B. Com. (Hons.) III Semester Examination, 2013

Paper Code: AS-2623 MODEL ANSWERS

Paper Title: Business Mathematics

1. (i) Let, $y = \log_{23} 243$

$$y = \log_{10} 243 \times \log_{23} 10$$

$$\text{or, } y = \log_{10} 243 \times \frac{1}{\log_{10} 23}$$

$$y = \frac{\log_{10} 243}{\log_{10} 23}$$

$$y = \frac{2.3856}{1.3617}$$

$$y = 1.75 \text{ approx.}$$

Ans.

(ii) Given, $A = \begin{bmatrix} 3 & 1 \\ 7 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 7 \\ 5 & 2 \end{bmatrix}$

So, $$A+B = \begin{bmatrix} 3+4 & 1+7 \\ 7+5 & 6+2 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 7 & 8 \\ 12 & 8 \end{bmatrix}_{2\times 2}$$

Ans.

(iii) Given, ratio is 5:6.

Let, the numbers be $5x$ and $6x$ respectively.

Now, as per question –

$$\frac{5x+5}{6x+5} = \frac{6}{7}$$

or, $7(5x+5) = 6(6x+5)$

$$35x+35 = 36x+30$$

$$35x-36x = 30-35$$

$$\cancel{-}x = \cancel{-}5$$

$$\boxed{x=5}$$

So, numbers are $\left.\begin{array}{l} 5x = 5\times5 = 25 \\ 6x = 6\times5 = 30 \end{array}\right\}$ Ans.

(iv) Let, the proportion between salary of A & B is $x:y$.

So, ratio between B & C' salary $= x:y$

It can be shown as –

$$\begin{array}{ccccc} A & : & B & : & C \\ x & : & y & & \\ & & x & : & y \end{array}$$

To obtain continued ratio between A, B and C ; Let it is $x:y:z$

i.e. $A : B : C$

$x : y : z$

$x : y$

or, $\frac{y}{x} = \frac{z}{y}$

or, $z = \frac{y^2}{x}$

Now, $x : y : z :: 160 : y : 250$ (as per question)

it means $x = 160$ ——— (i)

$z = 250$ ——— (ii)

$\because \quad z = \frac{y^2}{x} = 250$

$y^2 = 250 \times x$

Putting the value of x in above expression -

$y^2 = 250 \times 160$

$y = \sqrt{250 \times 160}$

$\cancel{= 50 \times 40}$

$= \sqrt{25 \times 16 \times 100}$

$= 5 \times 4 \times 10$

$y = ₹\ 200$

i.e., $\boxed{\text{Salary of } B = ₹\ 200}$ Ans.

(v) Cost Price = ₹ 1,250

Selling Price = ₹ 1,100

Loss = Cost Price – Selling Price

= 1,250 – 1,100

= ₹ 150

$$\text{Loss percent} = \frac{\text{Loss}}{\text{Cost Price}} \times 100$$

$$= \frac{150}{1,250} \times 100$$

$$\boxed{= 12\%}$$

Ans.

(vi) Let, $y = e^{3x}$

again, let $3x = t$

So, $y = e^t$

Differentiating above expression w.r.t. x at both sides –

$$\frac{d}{dx}(y) = \frac{d}{dx}(e^t)$$

$$\frac{dy}{dx} = \frac{d}{dx}e^t$$

Since, e^t cannot be differentiated w.r.t. x; So multiplying by dt at numerator and at denominator; we get –

$$\frac{dy}{dx} = \frac{d}{dx} e^t \frac{dt}{dt}$$

$$= \frac{d}{dt} e^t \frac{dt}{dx}$$

$$= e^t \frac{d}{dx}(t)$$

$$\frac{dy}{dx} = e^t \frac{d}{dx}(t)$$

Replacing the value of t in above expression –

$$\frac{dy}{dx} = e^{3x} \frac{d}{dx}(3x)$$

$$= e^{3x}\, 3 . \frac{d}{dx}(x)$$

$$= e^{3x}\, 3 . (1)$$

$$\boxed{\frac{dy}{dx} = 3e^{3x}}$$

Ans.

(vii) Given, $y = \frac{\log x}{x}$

Differentiating above function w.r.t. x –

$$\frac{dy}{dx} = \frac{x \frac{d}{dx}(\log x) - \log x \frac{d}{dx}(x)}{x^2}$$ [Applying division formula]

$$= \frac{x\left(\frac{1}{x}\right) - \log x (1)}{x^2}$$

$$\frac{dy}{dx} = \frac{1-\log x}{x^2} \qquad \text{— (i)}$$

Again, differentiating eq^{n}. (i) w.r.t. x –

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{x^2 \frac{d}{dx}(1-\log x) - (1-\log x)\frac{d}{dx}(x^2)}{(x^2)^2}$$

$$\frac{d^2y}{dx^2} = \frac{x^2\left[\frac{d}{dx}(1) - \frac{d}{dx}(\log x)\right] - (1-\log x)(2x)}{x^4}$$

$$= \frac{x^2\left(0 - \frac{1}{x}\right) - 2x + 2x\log x}{x^4}$$

$$= \frac{-x - 2x + 2x\log x}{x^4}$$

$$= \frac{2x\log x - 3x}{x^4}$$

$$\frac{d^2y}{dx^2} = \frac{\cancel{x}(2\log x - 3)}{x^{\cancel{4}3}}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{2\log x - 3}{x^3}}$$

Ans.

(viii) Given, $A = ₹\ 20,900$; $P = ?$

$R = 6\%$ p.a.

$T = 9$ months $= \frac{9}{12}$ year $= \frac{3}{4}$ year

$\because$ Simple interest $= \frac{PRT}{100}$

Putting the known values, $SI = \frac{P \times \cancel{6}^{3} \times 3}{\cancel{4}_{2} \times 100}$

$= \frac{9P}{200}$

$\because$ Amount = Principal + Simple Interest

$20,900 = P + \frac{9P}{200}$

$20,900 = \frac{209P}{200}$

or, $P = \frac{20,900 \times 200}{209}$

$\boxed{P = ₹\ 20,000}$ Ans.

(ix) Given, $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$

To find $\log \frac{26}{51} - \log \frac{91}{119}$

$= \log \frac{26/51}{91/119}$ $\left(\because \log \frac{m}{n} = \log m - \log n\right)$

$= \log \frac{\cancel{26}^{2} \times \cancel{119}^{7}}{\cancel{91}_{7} \times \cancel{51}_{3}}$

$= \log \frac{2}{3}$

$= \log 2 - \log 3$

$= 0.3010 - 0.4771$

$= -0.1761$ <u>Ans.</u>

(x) Row matrix is a type of matrix which has only one row. It can have any numbers of columns.
For e.g. – $A_{1\times4}$; $B_{1\times2}$; $C = \begin{bmatrix} 0 & 3 & -1 & 4 \end{bmatrix}_{1\times4}$

Long Answer Questions

2. Given, $y = \sqrt{\dfrac{1-x}{1+x}}$

or, $y = \dfrac{\sqrt{1-x}}{\sqrt{1+x}}$

or, $y = \dfrac{(1-x)^{1/2}}{(1+x)^{1/2}}$

Let, $(1-x) = u$ and $(1+x) = v$

So, $y = \dfrac{u^{1/2}}{v^{1/2}}$ ——— (i)

Differentiating eqn. (i) w.r.t. x —

$$\frac{d}{dx}(y) = \frac{v^{1/2}\frac{d}{dx}u^{1/2} - u^{1/2}\frac{d}{dx}v^{1/2}}{(v^{1/2})^2}$$ [Applying division formula]

$$\frac{dy}{dx} = \frac{v^{1/2} \frac{d}{dx} u^{1/2} \frac{du}{du} - u^{1/2} \frac{d}{dx} v^{1/2} \frac{dv}{dv}}{v}$$

$$= \frac{v^{1/2} \frac{d}{du} u^{1/2} \frac{du}{dx} - u^{1/2} \frac{d}{dv} v^{1/2} \frac{dv}{dx}}{v}$$

$$= \frac{v^{1/2} \frac{1}{2} u^{\frac{1}{2}-1} \frac{d}{dx}(u) - u^{1/2} \cancel{d} \frac{1}{2} v^{\frac{1}{2}-1} \frac{d}{dx}(v)}{v}$$

$$= \frac{v^{1/2} \frac{1}{2} u^{-1/2} \frac{d}{dx}(u) - u^{1/2} \frac{1}{2} v^{-1/2} \frac{d}{dx}(v)}{v}$$

$$= \frac{\frac{1}{2} \cdot \frac{v^{1/2}}{u^{1/2}} \cdot \frac{d}{dx}(u) - \frac{1}{2} \cdot \frac{u^{1/2}}{v^{1/2}} \cdot \frac{d}{dx}(v)}{v}$$

Putting the values of u and v back in above step —

$$= \frac{\frac{1}{2} \frac{(1+x)^{1/2}}{(1-x)^{1/2}} \frac{d}{dx}(1-x) - \frac{1}{2} \frac{(1-x)^{1/2}}{(1+x)^{1/2}} \frac{d}{dx}(1+x)}{(1+x)}$$

$$\frac{dy}{dx} = \frac{\frac{1}{2} \frac{\sqrt{1+x}}{\sqrt{1-x}} \left\{\frac{d}{dx}(1) - \frac{d}{dx}(x)\right\} - \frac{1}{2} \frac{\sqrt{1-x}}{\sqrt{1+x}} \left\{\frac{d}{dx}(1) + \frac{d}{dx}(x)\right\}}{(1+x)}$$

$$\frac{dy}{dx} = \frac{\frac{1}{2}\frac{\sqrt{1+x}}{\sqrt{1-x}}(0-1) - \frac{1}{2}\frac{\sqrt{1-x}}{\sqrt{1+x}}(0+1)}{(1+x)}$$

$$= \frac{-\frac{1}{2}\frac{\sqrt{1+x}}{\sqrt{1-x}} - \frac{1}{2}\frac{\sqrt{1-x}}{\sqrt{1+x}}}{(1+x)}$$

$$= \frac{-\frac{1}{2}\left\{\frac{\sqrt{1+x}}{\sqrt{1-x}} + \frac{\sqrt{1-x}}{\sqrt{1+x}}\right\}}{(1+x)}$$

$$= \frac{-\frac{1}{2}\left\{\frac{\sqrt{1+x}\sqrt{1+x}+\sqrt{1-x}\sqrt{1-x}}{\sqrt{1-x}\sqrt{1+x}}\right\}}{(1+x)}$$

$$= -\frac{1}{2}\frac{\left\{\frac{(1+x)+(1-x)}{\sqrt{(1-x)(1+x)}}\right\}}{(1+x)}$$

$$= -\frac{1}{2}\frac{\left\{\frac{1+\cancel{x}+1-\cancel{x}}{\sqrt{1-x^2}}\right\}}{(1+x)} \qquad \left[\because (1+x)(1-x) = (1-x^2)\right]$$

$$= -\frac{1}{\cancel{2}}\frac{\frac{\cancel{2}}{\sqrt{1-x^2}}}{(1+x)}$$

$$\boxed{\frac{dy}{dx} = -\frac{1}{(1+x)\sqrt{1-x^2}}}$$

Hence proved

3. (a) Let, $y = \dfrac{\sqrt{78.23} \times \sqrt[3]{0.024}}{(0.9694)^2}$

or, $y = \dfrac{(78.23)^{1/2} \times (0.024)^{1/3}}{(0.9694)^2}$

Taking log both sides —

$\log y = \log \dfrac{(78.23)^{1/2} \times (0.024)^{1/3}}{(0.9694)^2}$

$\log y = \log (78.23)^{1/2} + \log (0.024)^{1/3} - \log (0.9694)^2$

[by using $\log m \times n = \log m + \log n$
and $\log \frac{m}{n} = \log m - \log n$]

$\log y = \frac{1}{2} \log 78.23 + \frac{1}{3} \log 0.024 - 2 \log 0.9694$

[using $\log m^n = n \log m$]

Putting the log values from log table —

$\log y = \frac{1}{2} \times (1.8934) + \frac{1}{3} \times (\bar{2}.3802) - 2(\bar{1}.9865)$

$= \frac{1}{2} \times 1.8934 + \frac{1}{3}(-2 + 0.3802) - 2(-1 + 0.9865)$

$= 0.9467 + \frac{1}{3}(-1.6198) - 2(-0.0135)$

$= 0.9467 - 0.5399 + 0.027$

$\log y = 0.4338$

Taking Antilog both sides —

$y = A.L.\ 0.4338$

$$\boxed{y = 2.715}$$

Ans.

(b) Given, $p = 2\log_8 n$ —— (i)

$q = \log_2 2n$ —— (ii)

and, $q - p = 4$ —— (iii)

Putting the value of p and q in eq$^{n.}$ (iii) —

$\log_2 2n - 2\log_8 n = 4$

$\log_2 2n - \log_8 n^2 = 4$

$\log_2 2n - \log_{2^3} n^2 = 4$

By the property of log —

$\log_2 2n - \frac{1}{3}\log_2 n^2 = 4$

~~using $\log m - \log n = \log \frac{m}{n}$~~ —

$\cancel{\log \frac{2n}{n^2}}$

$$\log_2 2n - \log_2 (n^2)^{1/3} = 4$$

$$\log_2 2n - \log_2 n^{2/3} = 4$$

$$\log_2 \frac{2n}{n^{2/3}} = 4 \quad [\text{by } \log m - \log n = \log \tfrac{m}{n}]$$

With the help of exponential form —

$$2^4 = \frac{2n}{n^{2/3}}$$

$$2^4 = 2n . n^{-2/3}$$

$$2^4 = 2. n^{1-\frac{2}{3}}$$

$$\frac{2^4}{2} = n^{1/3}$$

$$n^{1/3} = 2^3$$

To remove the power $1/3$ from n, we have to put a power of 3 at both sides —

$$(n^{1/3})^3 = (2^3)^3$$

$$n^{\frac{1}{3}\times 3} = 2^{3\times 3}$$

$$n = 2^9$$

$$\boxed{n = 512}$$

Ans.

4. Given,

Class	First	Second	Third
Ratio in fares	10 :	8 :	3
Ratio of No. of passengers	3 :	4 :	10

Let, the fares be ₹ $10x$, ₹ $8x$, and ₹ $3x$ respectively and let the no. of passengers be $3y$, $4y$, and $10y$ respectively

Now, The total fare from a class = Fare from that class × No. of passengers of that class

So, for first class $= 10x \times 3y$

$= ₹\ 30xy$

for second class $= 8x \times 4y$

$= ₹\ 32xy$

and for Third Class $= ₹\ 3x \times 10y$

$= ₹\ 30xy$

Total Fare $= ₹\ 30xy + ₹\ 32xy + ₹\ 30xy = ₹\ 92xy$

When ~~total~~ total fare is ₹ $92xy$, share of 2^{nd} class fare = ₹ $32xy$

When total fare is ₹ $1xy$, share of 2^{nd} class fare $= \frac{₹\ 32xy}{₹\ 92xy}$

When total fare is ₹ 16,100 ; share of 2^{nd} class fare =

$$= \frac{32xy}{92xy} \times 16{,}100$$

$= ₹\ 5{,}600$

So, the amount that shall be realised by the sale of Second class tickets is ₹ 5,600. <u>Ans.</u>

<u>15.</u> In this question, time-period involved is 2 years while principal is not given. rate of borrowing and lending shall be applied accordingly.

<u>In the case of borrowing –</u>

Let, Principal (P) = ₹ 100

rate (r) = 10% per annum

Time (t) = 2 years

So, Amount (A) $= P\left(1+\frac{r}{100}\right)^t$

$= 100\left(1+\frac{10}{100}\right)^2$

$= 100 \times \frac{110}{100} \times \frac{110}{100}$

$= ₹\ 121$

Compound Interest = Amount (A) – Principal (P)

$= 121 - 100$

$= ₹\ 21$

In the case of lending

Principal $(P) = ₹\ 100$

rate $(r) = 10\%$ p.a.

$= \frac{10}{2} = 5\%$ per half-yearly

Time $(t) = 2$ years

$= 4$ half-years

So, Amount $(A) = P\left(1+\frac{r}{100}\right)^{t}$

$= 100\left(1+\frac{5}{100}\right)^{4}$

$= 100 \times \frac{105}{100} \times \frac{105}{100} \times \frac{105}{100} \times \frac{105}{100}$

$= ₹\ 121.55$

Compound Interest $=$ Amount (A) $-$ Principal (P)

$= 121.55 - 100$

$= ₹\ 21.55$

Gain $=$ Compound Interest earned by lending $-$ Compound interest paid at borrowing

$= ₹\ 21.55 - ₹\ 21.00$

$= ₹\ 0.55$

Now, by Unitary Method –

When gain is ₹ 0.55, Principal involved is = ₹ 100

$\therefore$ When gain is ₹ 1.00, Principal involved is $= \dfrac{₹100}{₹0.55}$

$\therefore$ When gain is ₹ 16.80, Principal involved is $= \dfrac{₹100}{₹0.55} \times ₹16.80$

$= ₹\, 3054.55$

So, Principal (or, sum) borrowed = ₹ 3,054.55

<u>Ans.</u>

6. Given, Demand function $p = 400 - 2x$

Average cost function $AC = 0.2x + 4 + \dfrac{400}{x}$

$$\because \quad AC = \frac{\text{Total Cost (TC)}}{x}$$

or, Total Cost (TC) = Average Cost (AC) × x

$$= \left[0.2x + 4 + \frac{400}{x}\right] \times x$$

$$TC = 0.2x^2 + 4x + 400$$

Tax per unit = ₹ 22

Total Tax = ₹ $22x$

This tax will be added on the total cost function.

So, $TC = 0.2x^2 + 4x + 400 + 22x$

$$TC = 0.2x^2 + 26x + 400 \quad \text{—— (i)}$$

Now, $\underset{(TR)}{\text{Revenue}} = \text{Price} \times x$

$$TR = (400 - 2x) \times x$$

$$TR = 400x - 2x^2 \quad \text{—— (ii)}$$

Again, $\text{Profit} = TR - TC$

Putting the values of TR and TC in above function –

$$\text{Profit} = 400x - 2x^2 - (0.2x^2 + 26x + 400)$$

$$\text{Profit} = 400x - 26x - 2x^2 - 0.2x^2 - 400$$

$$\text{Profit} = -2.2x^2 + 374x - 400 \quad \text{—— (iii)}$$

Mathematically, profit will be maximum where $\frac{d}{dx}(\text{Profit})$ attains zero (Maxima property) —

So, differentiating above eqn. (iii) w.r.t. output (x) —

$$\frac{d}{dx}(\text{Profit}) = \frac{d}{dx}\left\{-2.2x^2 + 374x - 400\right\}$$

$$= -2.2\frac{d}{dx}(x^2) + 374\frac{d}{dx}(x) - \frac{d}{dx}(400)$$

$$= -2.2(2x) + 374(1) - 0$$

$$\frac{d}{dx}(\text{Profit}) = 374 - 4.4x \quad \text{———— (iv)}$$

or, $374 - 4.4x = 0$

$$374 = 4.4x$$

or, $x = \frac{374}{4.4}$

$$x = 85$$

To confirm that at $x = 85$, function attains maxima, we have to differentiate eqn. (iv) w.r.t. x —

$$\frac{d}{dx}\left[\frac{d}{dx}(\text{profit})\right] = \frac{d}{dx}(374 - 4.4x)$$

$$\frac{d^2(\text{profit})}{dx^2} = \frac{d}{dx}(374) - 4.4.\frac{d}{dx}(x)$$

$$= 0 - 4.4.(1)$$

$$= -4.4$$

$\Rightarrow$ Negative value

So, it is clear that at output level $\boxed{x = 85}$; function attains maxima.

To identify the profit maximising price, putting $x = 85$ in the demand (or, price) function —

$$P = 400 - 2x$$

$$P = 400 - 2 \times 85$$

$$P = 400 - 170$$

$$\boxed{P = ₹\ 230}$$

So, Profit Maximising price = ₹ 230

Profit Maximising output = 85 units

Ans.

7. Given,

(i) Objective Function –

$$Z_{min.} = 2x_1 + 4x_2$$

(ii) Set of restrictions –

$$6x_1 + x_2 \geq 18$$

$$x_1 + 4x_2 \geq 12$$

$$2x_1 + x_2 \geq 10$$

(iii) Non-negative restriction –

$$x_1, x_2 \geq 0$$

Step – Plotting

(a) Restriction – 1

$$6x_1 + x_2 = 18$$

x_1	0	3
x_2	18	0

So, two points on graph will be (0,18) and (3,0). These two points form Line AF.

(b) Restriction 2

$$x_1 + 4x_2 = 12$$

x_1	0	12
x_2	3	0

So, two points on graph will be (0,3) and (12,0). These two points form line DE.

(c) Restriction 3

$$2x_1 + x_2 = 10$$

x_1	0	5
x_2	10	0

So, two points on graph will be (0,10) and (5,0)
These two points form Line GH.

Step - Determination of Solution region

- Line GH and AF cuts each other at Point B which is at (2,6) position.
- Line GH and DE cuts each other at point C which is at (4,2) position.

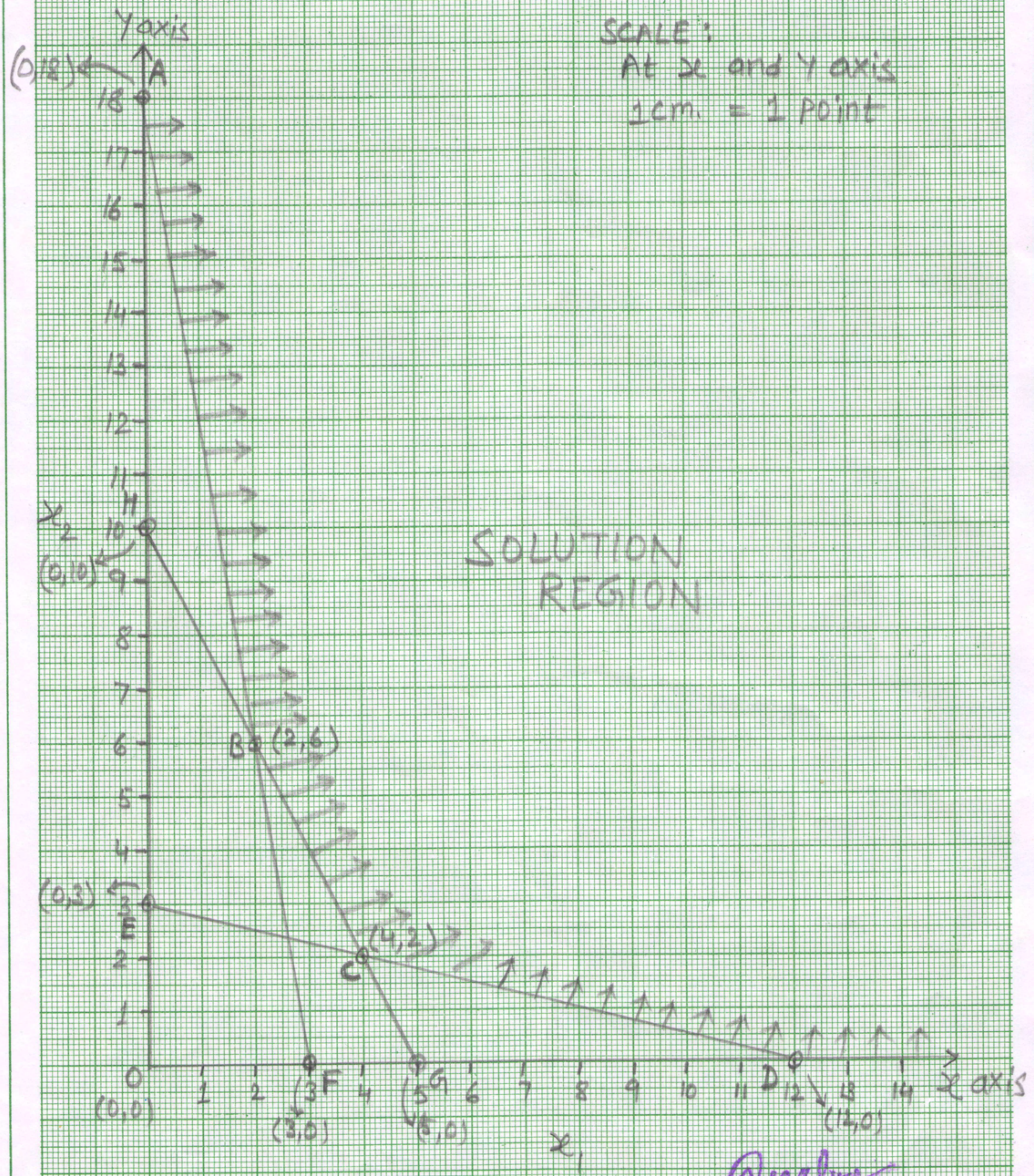
Since, this is Cost-minimising problem, the Solution region will lie beyond origin O and beyond point B and C.

So the solution region will be determined at Region beyond point A, B, C and D.

Step - Determination of Corner points

The determined Solution region has following corner points:

A (0,18) ; B (2,6) ; C (4,2) ; and D (12,0)



Y axis
SCALE :
At x and Y axis
1cm = 1 Point
(0,18)
A
H
x2
(0,10)
B
(2,6)
SOLUTION
REGION
(0,3)
E
(4,2)
C
F
G
D
(0,0)
(3,0)
(5,0)
(12,0)
x axis
x1

Step Determination of Minimum Cost

Corner Point	Objective Function $Z_{min.} = 2x_1 + 4x_2$	Minimum Cost $Z_{min.}$
A (0,18)	$= 2\times0 + 4\times18$	72
B (2,6)	$= 2\times2 + 4\times6$	28
C (4,2)	$= 2\times4 + 4\times2$	16
D (12,0)	$= 2\times12 + 4\times0$	24

At corner point C(4,2), there will be minimum cost and it will be 16 units.

Ans.

8. From the given question, three matrices can be obtained. These are

Per unit product/labour requirement matrix (Let, X)

$$X = \begin{bmatrix} 50 & 100 & 30 \\ 30 & 50 & 20 \\ 20 & 40 & 10 \end{bmatrix}_{3\times3}$$

Cost of Material/labour (say, Y) —

$$Y = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}_{3\times1}$$

Per unit cost can be obtained by multiplying X and Y

$$X.Y = \begin{bmatrix} 50 & 100 & 30 \\ 30 & 50 & 20 \\ 20 & 40 & 10 \end{bmatrix}_{3\times3} \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}_{3\times1}$$

$$X.Y_{3\times1} = \begin{bmatrix} 50\times10 + 100\times20 + 30\times30 \\ 30\times10 + 50\times20 + 20\times30 \\ 20\times10 + 40\times20 + 10\times30 \end{bmatrix}_{3\times1}$$

$$XY = \begin{bmatrix} 500 + 2{,}000 + 900 \\ 300 + 1{,}000 + 600 \\ 200 + 800 + 300 \end{bmatrix}$$

$$XY = \begin{bmatrix} 3{,}400 \\ 1{,}900 \\ 1{,}300 \end{bmatrix}_{3\times1}$$

Another matrix is Z which is showing different units of tables produced —

$$Z = \begin{bmatrix} 100 & 200 & 300 \end{bmatrix}_{1\times3}$$

So, Total Cost can be obtained by multiplying Z with XY —

$$Z.XY = \begin{bmatrix} 100 & 200 & 300 \end{bmatrix}_{1\times3} \begin{bmatrix} 3{,}400 \\ 1{,}900 \\ 1{,}300 \end{bmatrix}_{3\times1}$$

$$ZXY \text{ (or, Total Cost)} = [100 \times 3{,}400 + 200 \times 1{,}900 + 300 \times 1{,}300]$$

$$= [3{,}40{,}000 + 3{,}80{,}000 + 3{,}90{,}000]$$

$$\boxed{\text{Total Cost} = ₹\ 11{,}10{,}000}$$

<u>Ans.</u>

— x —